Closing Tues: HW 9.7(2), 9.8, 9.9

Exam 1 is Thur, Jan. 25th covers 9.3 - 9.9.

9.9 Applications *(continued)*

Recall from last class (and Math 111):

If $p(x) =$ selling price per item (demand)

$AC(x) =$ average cost per item

then
$$TR(x) = p(x) \cdot x$$
$$TC(x) = AC(x) \cdot x.$$

Entry Task (*directly* from HW 9.9/5)

The price of a certain product is \$400.

The cost per unit of producing the product is 130 + 0.5x dollars/item.

Give the functions *TR(x), TC(x), MR(x)* and *MC(x)*.

$TR(x) = p \cdot x = 40$

$\Rightarrow \boxed{\begin{array}{l} TR(x) = 400x \\ MR(x) = TR'(x) = 400 \end{array}}$

$TC(x) = AC(x) \cdot x = (130 + 0.5x)x$

$\Rightarrow \boxed{\begin{array}{l} TC(x) = 130x + 0.5x^2 \\ MC(x) = TC'(x) = 130 + x \end{array}}$

Recall from Math 111:

Profit and marginal profit are given by

$$P(x) = TR(x) - TC(x)$$

$$MP(x) = MR(x) - MC(x)$$

When profit is maximized

$$MR(x) = MC(x)$$

Specifically, where it switches from MR > MC to MR < MC.

$$P(x) = TR(x) - TC(x)$$

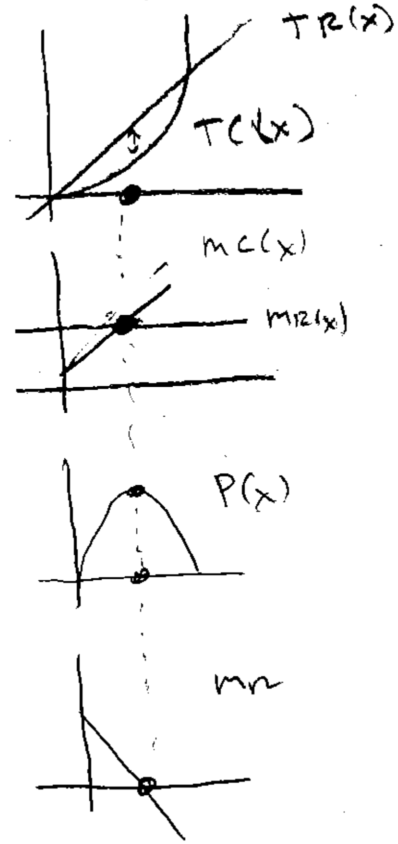
$$= (400x) - (130x + 0.5x^2)$$

$$= 400x - 130x - 0.5x^2$$

$$\boxed{\begin{aligned} P(x) &= 270x - 0.5x^2 \\ MP(x) = P'(x) &= 270 - x \end{aligned}}$$

Continuing example from the entry task:

How many units should the firm produce and sell to maximize its profits?



$$MR(x) = MC(x)$$

$$400 = 130 + x$$

$$\Rightarrow \boxed{x = 270 \text{ items}}$$

$$MP(x) = 0$$

$$\Rightarrow 270 - x = 0$$

$$\Rightarrow \boxed{x = 270 \text{ items}}$$

occurs here

$$P(270) = 270(270) - 0.5(270)^2$$

$$= \boxed{\$36{,}450} \leftarrow \text{MAX PROFIT}$$

Another example
(*directly* from an old midterm):

You sell items.
If q is in **hundred items**, then TR(q) and TC(q) in **hundred dollars** are given by

$$TR(q) = 30q$$
$$TC(q) = q^3 - 15q^2 + 78q + 10$$

a. Find marginal cost at 2 hundred items

$MC(q) = TC'(q) = 3q^2 - 30q + 78$

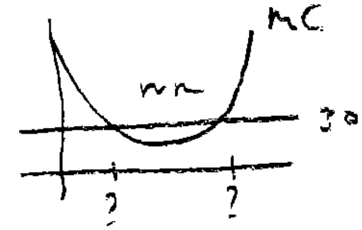
$MC(2) = 3(2)^2 - 30(2) + 78$

$= \boxed{30}$ $\boxed{\$/\text{item}}$

"The 201st item costs about \$30 to produce."

b. Find the longest interval over which marginal revenue exceeds marginal cost. WANT TO KNOW WHEN $MR(x) > MC(x)$

$\boxed{\text{STEP 1}}$ FIND MR = MC



$30 \stackrel{?}{=} 3q^2 - 30q + 78$

$\Rightarrow 0 = 3q^2 - 30q + 48$

$0 = q^2 - 10q + 16 = (q-2)(q-8)$

$q = 2$ OR $q = 8$ } OR USE QUAD. FORMULA TO GET THESE

$\boxed{\text{STEP 2}}$ INTERPRET

$\boxed{\text{MR is above MC between } q = 2 \text{ AND } q = 8}$

NOTE: PROFIT IS <u>INCREASING</u> FROM $q = 2$ to $q = 8$, SO

$\boxed{\text{MAX PROFIT OCCURS AT } q = 8}$

c. What is the maximum value of profit?

$P(8) = TR(8) - TC(8)$

$TR(8) = 30(8) = 240$

$TC(8) = (8)^3 - 15(8)^2 + 78(8) + 10$

$= 186$

$P(8) = 240 - 186$

$= 54$ hundred dollars

($5,400)

Graphs and Derivatives

Example: Let $f(x) = 2x^2 - 3x$

Find $f'(x)$. $= 4x - 3$

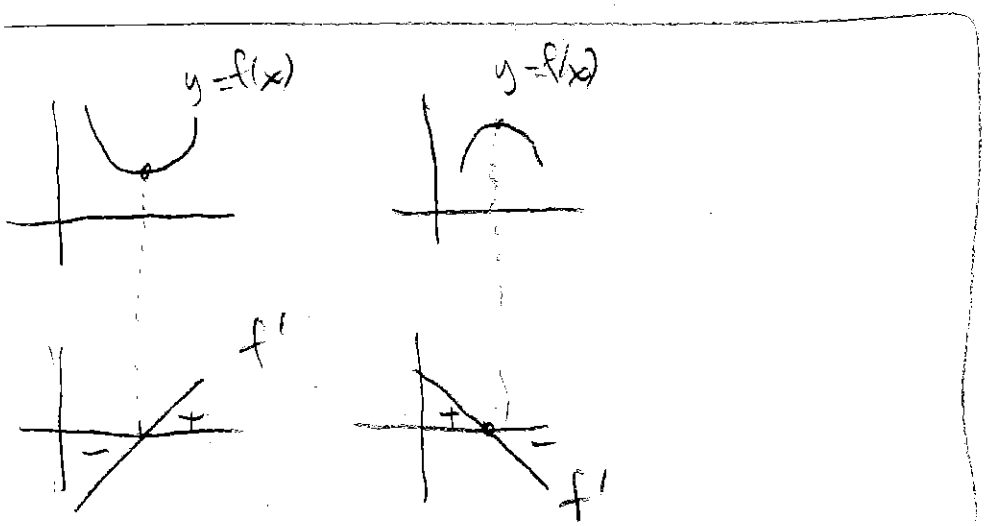
Note:

$\boxed{\text{I}}$ $f'(x) = 0$ WHEN

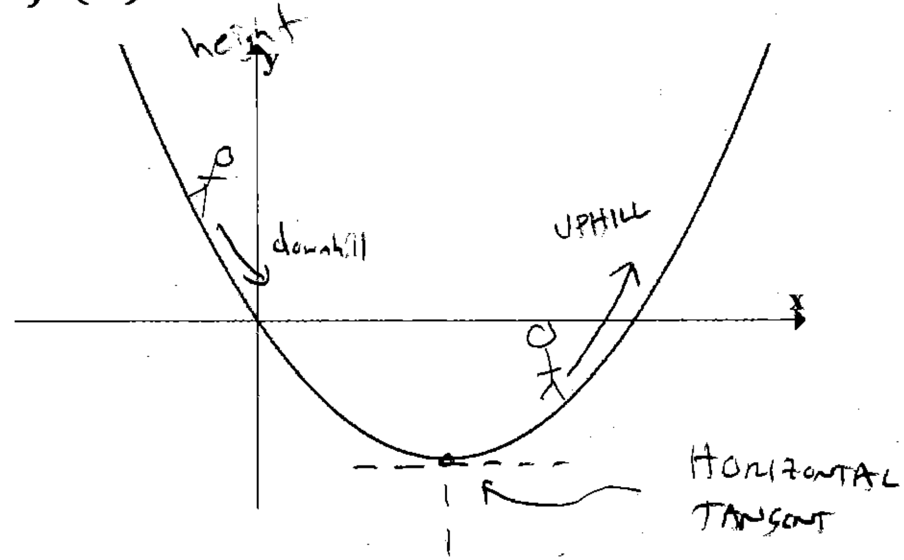
$4x - 3 = 0 \iff x = 3/4$

$\boxed{\text{II}}$ f' IS <u>POSITIVE</u> FOR ALL $x > 3/4$

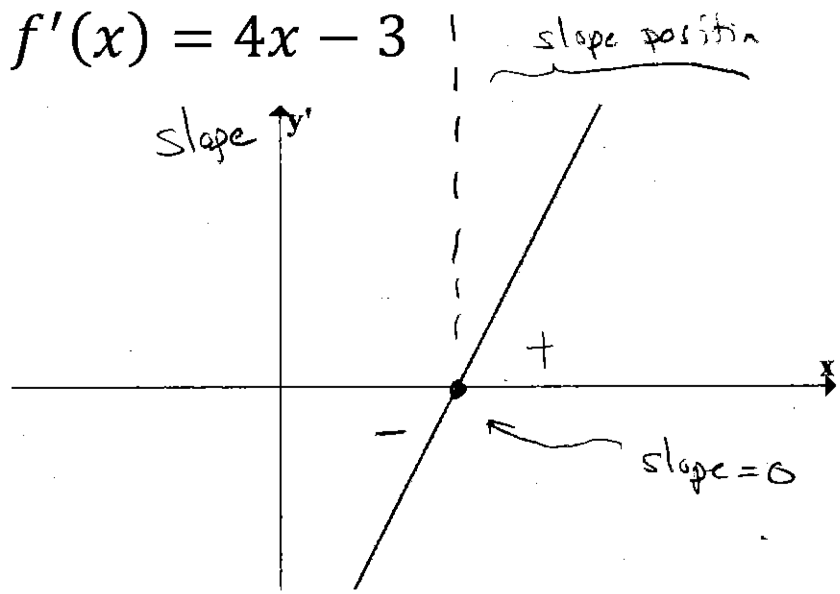
$\boxed{\text{III}}$ f' IS <u>NEGATIVE</u> FOR ALL $x < 3/4$



$f(x) = 2x^2 - 3x$



$f'(x) = 4x - 3$



Notes/Observations: Given $y = f(x)$.

- $y = f'(x)$ is a new function.
- $f(x) =$ "height of the graph at *x*"
- $f'(x) =$ "slope of $f(x)$ at x"
- $f'(x)$ is "instantaneous rate of change" (speedometer speed)
- The units of $f'(x)$ are $\frac{\text{y−units}}{\text{x−units}}$.

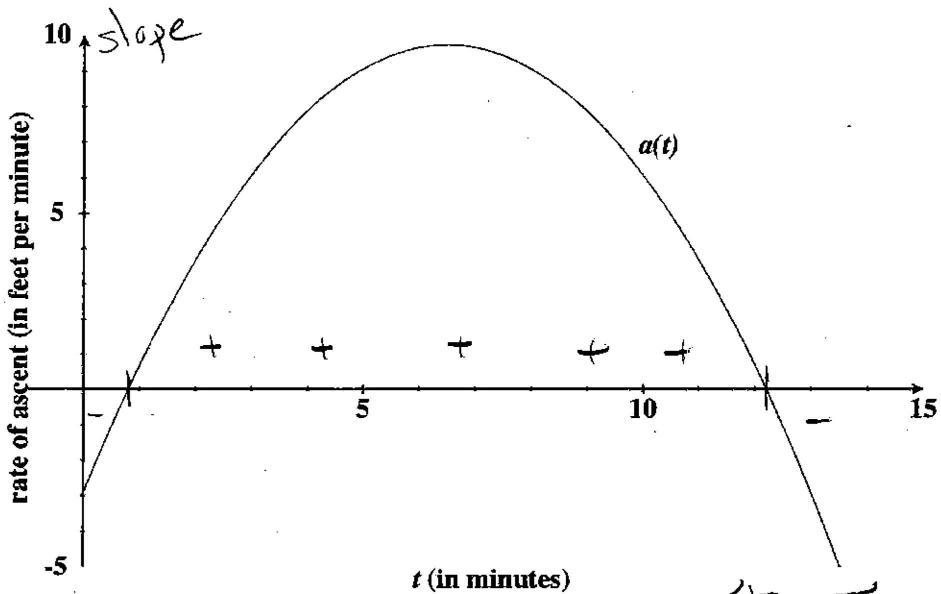
Fundamental to all applications:

$f(x)$	$f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative

9.9 HW Problem 8:

Rate of ascent for a balloon (in feet per minute) is given by

$$a(t) = -0.3t^2 + 3.9t - 2.928$$



FALLING RISING FALLING

How will you answer these:

(a) Find the longest integral over which Balloon A is rising.

(d) Find the time at which the balloon is rising the fastest?

(a) NEED TO FIND WHEN $a(t)$ IS POSITIVE.

$\boxed{\text{STEP 1}}$ FIND WHERE $a(t) \stackrel{?}{=} 0$.

$$-0.3t^2 + 3.9t - 2.928 \stackrel{?}{=} 0$$

QUAD. FORMULA

$\boxed{\text{STEP 2}}$ INTERPRET BETWEEN THESE

(d) WANT TO FIND HIGHEST POINT ON $a(t)$ (HORIZ. TANGENT)

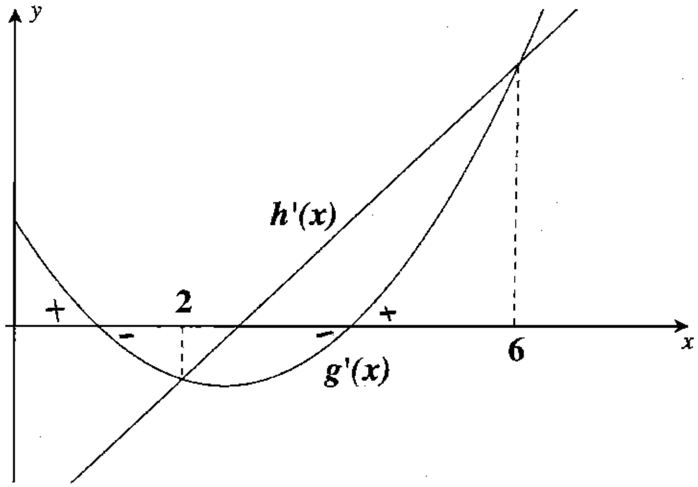
$\boxed{\text{STEP 1}}$ $a'(t) = -0.6t + 3.9$

$\boxed{\text{STEP 2}}$ Solve $a'(t) \stackrel{?}{=} 0$

HW 9.9/1:

Given $g'(x) = 2x^2 - 10x + 8$

$h'(x) = 6x - 16$



What does it mean when…

(a) … g'(x) crosses the x-axis?

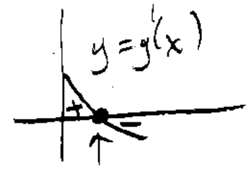
… h'(x) crosses the x-axis?

(b) … g'(x) has a horizontal tangent (and how do you find it)?

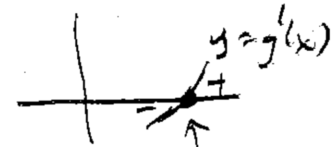
(c) … h'(x) intersects g'(x)?

(a)

$g'(x) \stackrel{?}{=} 0 \Leftrightarrow g(x)$ has a horizontal tangent



CORRESPONDS TO A MAX



CORRESPONDS TO A MIN.

$h'(x) = 0 \Leftrightarrow h(x)$ has a horiz. tangent

CORRESPONDS TO A MIN.

(b) g' has a horiz. tangent $\Leftrightarrow g''(x) \stackrel{?}{=} 0$

$4x - 10 \stackrel{?}{=} 0$

(c) $g'(x) = h'(x) \Leftrightarrow$ SAME SLOPE (RATE)

$\Leftrightarrow$ g(x) AND h(x) BIGGEST GAPS